AP Calculus

Summer Packet

Dear Prospective Calculus Students,

Welcome to AP Calculus. This is a rigorous, yet rewarding, math course. Most of the students who have taken Calculus in the past are amazed at how much they have to rely on prior knowledge from Algebra I and II to complete a problem. Many times they find that it is not the Calculus steps that trip them up, but the embedded Algebra that needs to be done. To help you prepare for this course and make it through the "tedious algebra", we feel that it would be beneficial to show you some of the skills you will come across and have you practice them.

You will also have to develop your critical thinking skills in this class. For some of you, this is a natural skill, for others, not so natural. There are many problems in this packet that don't "look" like something you know how to do, but are not that bad once you figure out what it's asking. This is typical of the type of problems that you will see in Calculus next year and we want to give you a chance to start practicing your problem solving skills early.

In addition to reviewing some of the past material learned, we need to get a jump start on the new material. In order to get everything taught in time, we would like to have you start to learn the first topic in Calculus – Limits. We have shown you examples and given you a few to try on your own. If you need further assistance, there are many web sites that have tutorials over Calculus material. One that we recommend is www.calculus-help.com. It has animated pictures of what is happening as you take a limit. You were also introduced to the topic in PreAP PreCal, although it may not have been called "limits".

You should do this packet without a calculator unless indicated. The answers are given so that you may check you work. Also, round everything to three decimal places.

In order to ensure that you complete the packet, we will be testing over this material sometime during the first two weeks of school. You will have a chance to get your questions answered the first week of school.

Have fun this summer, but do a little bit of studying. Believe us, a little studying now will save a lot of time this coming school year. We look forward to meeting all of you in the fall.

Patrick Dunn

Part I – Algebra Skills

Simplify the following.

1. $\dfrac{5(x+h)^3-5x^3}{h}$

2. $\dfrac{(x-1)^2(3x-1)-2(x-1)\bullet 3}{(x-1)^4}$

3. $\dfrac{(a/b)-a}{a+(a/b)^2}$

4. $\dfrac{2x(x+1)^2-3(x+1)^3}{8x^3+14x^2+6x}$

Solve for x.

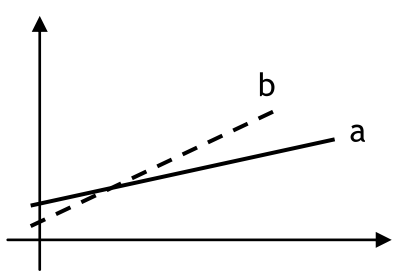
5. $\dfrac{3x+5}{(x-1)(x^4+7)}=0$

6. $(2x+1)(x-1)^2+(x+5)(2x+1)^2=0$

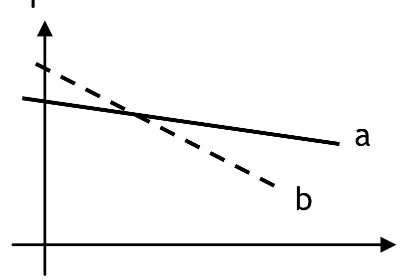
7. Solve for y_1: $xy_1+xy_1y^2-3=5y_1+xy$

8. Which of the following lines have a greater slope?

a.



b.



Write the expression as a sum of terms.

9. $\dfrac{u+1}{u}$

10. $\dfrac{u^{1/2}+u^{1/3}+1}{\sqrt{u}}$

11. $\dfrac{5x^3-2\sqrt{x}-4}{\sqrt{x}}$

Part II – Functions

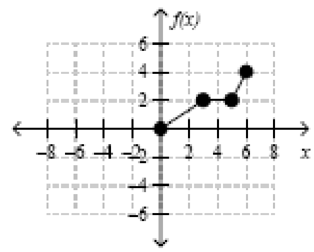
Refer to these tables of values to answer questions 12 - 14.

x	$f(x)$
1	4
2	3
3	5
4	2
5	1

x	$g(x)$
1	3
2	5
3	2
4	1
5	4

12. Find $g\left(f(3)\right)$.
13. Find $g^{-1}\left(f^{-1}(1)\right)$
14. Given $g(x)=-3f(x+1)-2x+3$ find $g(2)$

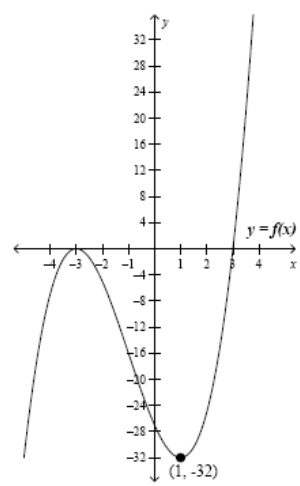
For questions 15 – 18, given the graph of $y=f(x)$ below, graph the following transformations.



15. $f(2x)$
16. $f\left(\frac{1}{2}x\right)$
17. $f(-x)$
18. $|f(x)|$

19. (Calc) Let $f(x)=7.364-1.793x$ and let $g(x)$ be a translation of $f(x)$. If the range of $g(x)$ is [-4.601, 6.157] on the domain [-1, 5], then what is $g(x)$ in terms of $f(x)$?

The graph of $y=f(x)$ shown is tangent to the $x-$axis at $x=-3$, has a zero at $x=3$ and a relative minimum at $x=1$. Use the graph of $y=f(x)$ shown to answer questions 20 – 22.



20. Write a transformation of $f(x)$ that will make the relative minimum have coordinates (5, -35)?

21. Choose all of the following equation(s) that would have a relative minimum at (1, -8).

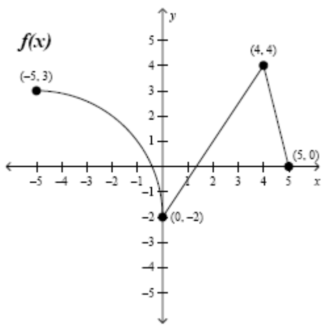
$g(x)=\frac{1}{4}f(x)$ $\qquad h(x)=-f(x-4)-8$

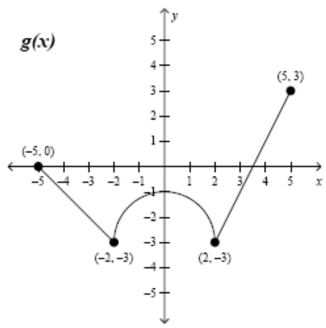
$m(x)=f(x)-8$

22. Choose the transformation that would make $f(x)$ have no real zeros?

$f(|x|)+2 \qquad f(x)+33 \qquad |f(x)|+2 \qquad -f(x)+33 \qquad f(x+3)+33$

23.Given the graphs of $f(x)$ and $g(x)$ shown above, if $h(x)=g(f(x))$, what is the range of $h(x)$?





24.(Calc)If $f(x)=x^{\frac{3}{2}}-1$, find the solution to the equation $f(x)=f^{-1}(x)$.

25.Given the function $f(x)=\begin{cases}2x+5 & x<0\\ -2x+5 & x\geq 0\end{cases}$, write an absolute value equation that is equivalent to $f(x)$.

Use the function $h(x)=\begin{cases}x^2-4x+3 & x<3\\ 2x-9 & x\geq 3\end{cases}$ *to answer questions 26-27*

26.What is the minimum value of $h(x)$?

27.For what values of x is $h(x)$ =8?

28.A swimming pool can hold a maximum of 360 gallons of water. The full pool develops a leak and is losing water at a constant rate. After 3 hours the pool has 354 gallons of water in it.

a. Write a function $g(t)$ for the total number of gallons of water that is in the pool in terms of the time, t, the number of hours since the pool developed the leak.
b. Find $g(20)$. Explain the meaning of the answer in the context of the problem.
c. If the leak is fixed after 20 hours and the owner immediately begins to fill the pool at a rate of 4 gallons of water per hour, write a piecewise function, $f(t)$, for the total number of gallons of water that is in the pool in terms of time, t.

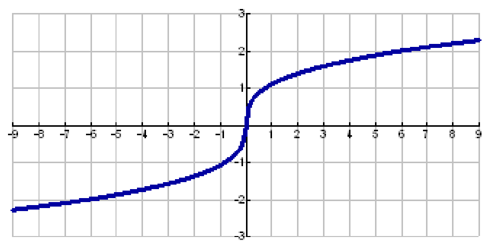
29. $f(x)=-|x+3|-2$

a. Graph $f(x)$
b. Domain: ___________ Range: ____________
c. $f(3)=$______________
d. If $f(x)=-3$, then $x=$__________
e. Rewrite without absolute values.

30. $g(x)=\begin{cases} \frac{x}{2}, & if\ x\ge 4 \\ \sqrt{x}, & if\ 0<x<4 \\ x^2, & if\ x<0 \end{cases}$

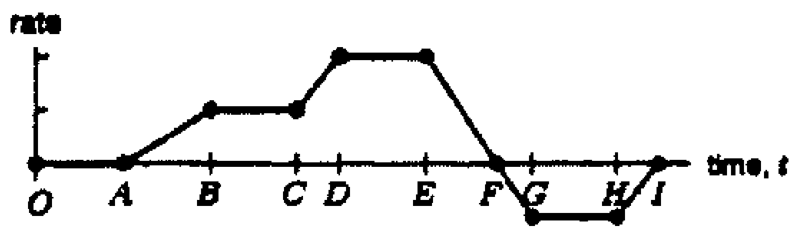
a. Graph $g(x)$
b. $g(-3)=$__________ $g(1)=$____________ $g(0)=$____________
c. Is $g(x)$ continuous?

31. Given the graph of $g(x)$ on the right.



a. Estimate $\frac{g(6)-g(0)}{6-0}$
b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch in this line segment.
c. Estimate the slope of the graph at the point (0, 0). Draw in this line.

32. The rate at which water is entering a tank $(t>0)$ is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:



a. The volume of water is constant.
b. The volume of water is decreasing.
c. The volume of water is increasing.
d. The volume of water is increasing fastest.

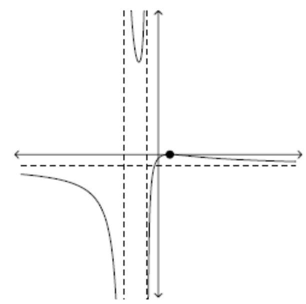
<u>Part III – Review of Polynomial and Rational Functions</u>

33. $Q(x)=\frac{3x}{x+1}$

a. Where is this function discontinuous?
b. State the equation of the vertical asymptote.
c. State the equation of the horizontal asymptote.
d. Sketch.

34. Find a value of p in the inequality $p+x-2x^2\ge 4$ if the solution is $-1\le x\le \frac{3}{2}$.

35. Write an equation of the function below if the vertical asymptotes are $x=-3$ and $x=-1$, horizontal asymptote $y=-1$ and $x-$intercept of 1.



36. Given $f(x)=2x^2-3x+2$, find the value of x that gives a slope of zero for the secant line passing through the two points $(x, f(x))$ and $(x+3, f(x+3))$.

37. Let $f(x)=\dfrac{x^2-a}{b(x-c)(x-d)}$. Find values of a, b, c and d such that $f(x)$ would have $x-$intercepts at ± 4, vertical asymptotes $x=9$ and $x=-2$, and end behavior asymptote $y=\dfrac{1}{3}$.

Use the given the rational function $f(x)=-2+\dfrac{x^2+x-5}{x-2}$ *to answer 38 - 41.*

38. Write the equation of the slant asymptote on the graph of $f(x)$.

39. (Calc) At what point on the graph does $f(x)$ have a relative minimum?

40. If $g(x)=f(x)(x-2)$, find the zero(s) of $g(x)$.

41. If $h(x)=f(x)+c$, find the value of c needed to make the relative minimum value of $h(x)$ equal to the relative minimum of $g(x)$.

42. Given the function, $h(x)=\dfrac{(ax-2)(x+b)}{(x+b)(x-c)}$. Which of the following statement(s) are true?

I. The function has a vertical asymptote at $x=c$.
II. the function has a vertical asymptote at $x=-b$.
III. The function has a horizontal asymptote at $y=a$.

43. What is the slant asymptote of the function $g(x)=\dfrac{3x^3+4x^2-x+2}{x^2-x-1}$?

Part IV – Review of Exponential and Logarithmic Functions

44. Sketch a graph of $y = e^x$ and $y = \ln x$.

45. Evaluate the following.

a. $\log_2 16$ b. $\log_3 1$ c. $\log 10$

d. $\ln 1$ e. $\ln e$ f. $\ln e^3$

46. Write an equation for the inverse of $f(x) = e^{2x-2}$.

47. Which of the following would be equivalent to $\frac{5}{2}\ln\left(4x^2\right)$ for $x > 0$?

I. $\ln 2 + \ln x$ II. $\ln 32 + \ln x^5$ III. $5\left[\ln 2 + \ln x\right]$

48. If $f(x) = 3^x$, then what is $f(b+3)$.

49. Graph $g(x) = 3\ln\left(2x\right)$ using transformations (without a calculator).

50. (Calc) Let $g(x) = 4xe^{4x}$, what are the coordinates of the absolute minimum point of $g(x)$?

Part V – Rates of Change

51. Consider the curve $x^2 + 4y^2 = 7 + 3xy$. The slope of the curve at any point on the graph can be found using $m = \dfrac{3y - 2x}{8y - 3x}$. Find a point on the curve at which the slope is 0.

52. For the curve $3y^2 - x + 2y = 12$, the slope function m for any point on the curve is given by $m = \dfrac{1}{6y + 2}$. Find the point on the curve where the slope is undefined.

53. A particle moving in a straight line has a velocity given by $v(t) = 2\sin(t) - 1$. What is the rate of change of velocity, called the average acceleration, of the particle over the interval $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$?

Part VI – Review of Trig

54. Evaluate without the use of a calculator.

a. $\tan\left(\frac{\pi}{6}\right)$ b. $\cos\left(\frac{-\pi}{3}\right)$ c. $\sin\left(\pi\right)$

d. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ e. $\cos^{-1}\left(\frac{1}{2}\right)$ f. $\arctan\left(-\sqrt{3}\right)$

55. Determine the domain, range, and period for six parent trig functions.

56. Of the six trig functions, which ones are even? Odd?

57. What is the domain of the function $f(x) = \arcsin\sqrt{x+2}$?

58. The equation $v(t) = 2\sin t + 1$ represents the velocity (rate of change of position) in feet per second at any time $t \geq 0$ of a particle's movement along a horizontal line. At which time(s) would the particle be at rest (a particle is at rest when it's velocity is zero).

59.(Calc) A particle moves along a horizontal axis so that its velocity in feet per second at any time $t \geq 0$ is given by $v(t) = 1 + 5\cos\left(\frac{t}{2}\right)$. When the velocity is negative, the particle is moving to the left. Find the intervals of t, $0 \leq t \leq 10$, when the particle is moving left.

60. For what value of a is $\sin(x+a) = \cos x$?

61. Graph the following $f(x) = 3\cos 4\left(x - \frac{\pi}{2}\right) - 2$.

Use the functions below to answer questions 62-65.

$f(x) = \cos^2 x$ $\qquad g(x) = \frac{1}{2}\sin x + \frac{1}{2}$ $\qquad h(x) = \frac{\cos(2x)+1}{2}$

62. If $f(1.5) = 0.005$, what is the value of $f(-1.5)$.
63. What is the maximum value attained by $g(x)$? If the domain of g is restricted to [0, 2π], when will $g(x)$ obtain the maximum value?
64. Find all values of x in [0, 2π] such that $f(x) = g(x)$?
65. Simplify $h(x)$ to show that $h(x) = f(x)$ for all real values of x.

Part VII – An Introduction to Limits

The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. We write $\lim_{x \to a} f(x)$, which is read "the limit of $f(x)$ as x approaches a".

The limit must be the same as x approaches "a" on both the left and the right.
There are many ways to find a limit. We will focus on three main ways for now: from a graph, from a table, and by direct substitution. Study the following examples of each type and then try the sample exercises on your own.

To find the limit from a table, look at the y-values as the x values get closer and closer to your "a" value. See if there is one number that all y-values seem to be going towards.

Example: Find $\lim_{x \to 2}(x^2)$.

X	1.9	1.99	1.999	2.001	2.01	2.1
Y	3.61	3.9601	3.996	4.004	4.0401	4.41

$\longrightarrow 4 \longleftarrow$

Solution: We are looking for the y-value as our x-values get closer and closer to 2. Looking at the chart from both the left and right sides, we can see that our y-values are getting closer and closer to 4. Therefore, $\lim_{x \to 2}(x^2) = 4$.

Your turn: Find the following limits using the charts.

1. $\lim_{x \to 3}(x^2 - 1)$.

X	2.9	2.99	2.999	3.001	3.01	3.1
Y	7.41	7.9401	7.994	8.006	8.0601	8.61

2. $\lim_{x \to -1}(2x)$.

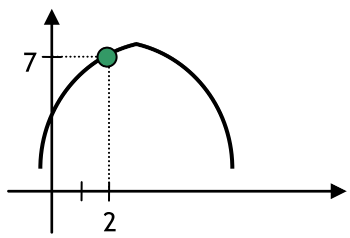
X	-0.9	-0.99	-0.999	-1.001	-1.01	-1.1
Y						

Solution: 1. 8; 2. -2

To find a limit from a graph, we follow the graph on either side of our "a" value towards that "a" value. The answer will be whatever y-value our graph is approaching. It is important to note that the graph does not have to actually "hit" that y-value. A limit is simply "what the y-value should be." If there is no clear y-value that your graph is approaching, or if there are two different y-values that your graph is approaching, then the limit does not exist (DNE).

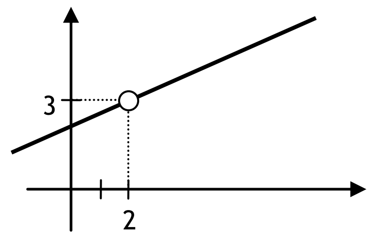
Example: Find the limit as x approaches 2 for each of the graphs below.

a.



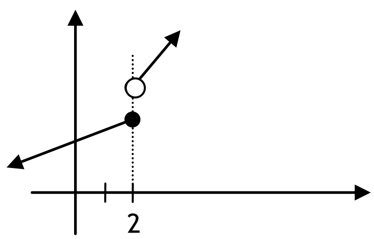
$\lim_{x \to 2} f(x) = 7$

b.



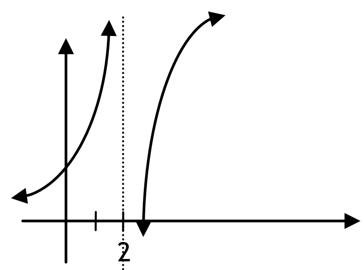
$\lim_{x \to 2} f(x) = 3$

c.



$\lim_{x \to 2} f(x)$ does not exist

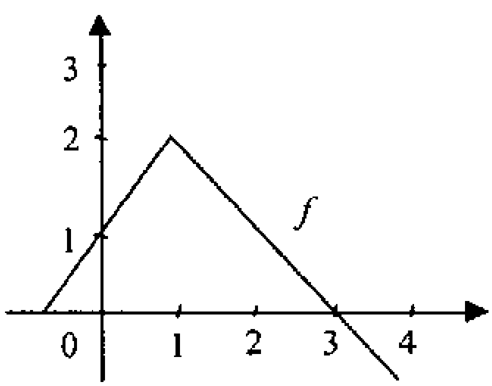
d.



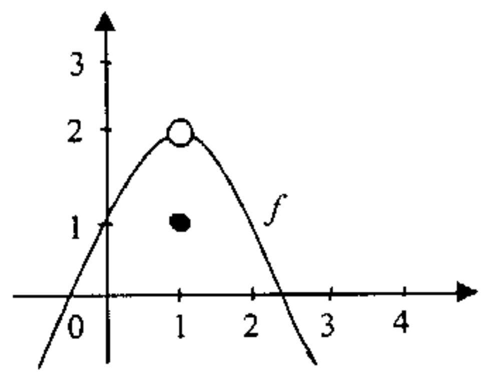
$\lim_{x \to 2} f(x)$ does not exist

Your turn: The graphs of some functions are pictured below. Do you think that $\lim_{x \to 1} f(x)$ exists? If so, state its value.

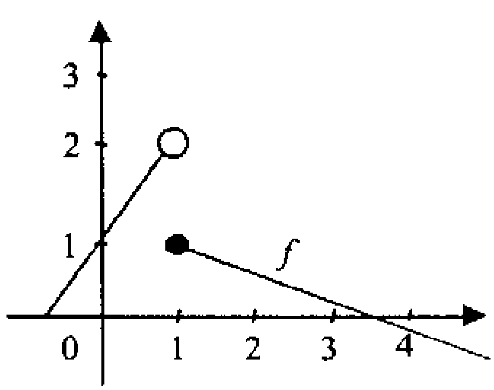
1.



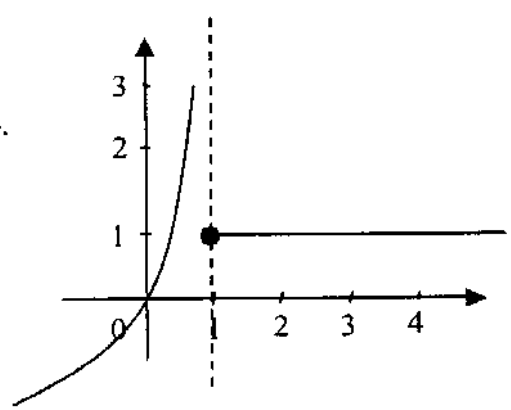
2.



3.



4.



Solution: 1. 2; 2. 2; 3. DNE; 4. DNE

To find a limit algebraically, you can use direct substitution. Simply plug the "a" value into the equation and simplify. As long as you do not get 0 on both the top and bottom (if it is a fraction), the number you get is the limit.

Example: Find the limit of the following functions.

$$\lim_{x\to 3}(3x-1)$$

Solution: We are looking for the limit as x approaches 3, so plug 3 into the expression $3x-1$. You get 8 as an answer. Therefore, $\lim_{x\to 3}(3x-1)=8$.

Your turn: Find the following limits.

1. $\lim_{x\to 2}(x^2-1)$ 2. $\lim_{x\to -4}(2x^2-6x+1)$ 3. $\lim_{x\to \pi}(\cos x)$

Solution: 1. 3; 2. 57; 3. -1

BC Only

Part I - Logistic Functions

1. (Calc) The red wolf, which once ranged naturally over the southern United States, from Texas to Florida, has been declared extinct in the wild. Environmentalists have reintroduced a small red wolf population into the Great Smoky Mountains National Park in Tennessee, and they projected that the size of the population in the park can be modeled with the logistics growth curve, $W(t)$, given below.

$W(t)=13+\dfrac{75}{1+24e^{(-0.4t)}}$, where t is time in years and $t=0$ represents January 1, 2005.

a. What is the initial population of red wolves in the Great Smoky Mountains National Park? During what year will the population of wolves reach 60? Express answers to the nearest whole number of wolves.

b. Find the $\lim_{x\to\infty} W(t)$ (or the horizontal asymptote) and explain its significance in terms of the population of wolves.

c. The population of red wolves will be growing at the fastest rate when it reaches half of its carrying capacity. The carrying capacity is the maximum number of wolves that the park can maintain. In what year will the population be growing at the fastest rate?

Part II - Sequences and Series

2. Choose which of the following series converges.

I. $1+3+5+\ldots$ II. $4+1+\frac{1}{4}+\ldots$ III. $\frac{1}{3}+\frac{2}{3}+\frac{4}{3}+\ldots$

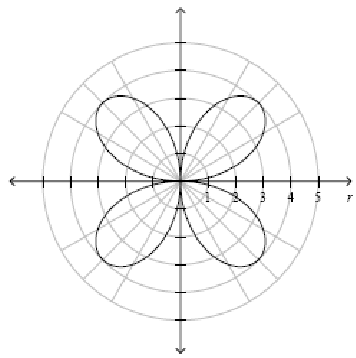
3. $\sum_{b=0}^{\infty} 9\left(\frac{2}{3}\right)^{b} =$

4. Which of the following could be the sigma notation for $\frac{1}{2}+\frac{2}{3}+\frac{6}{4}+\frac{24}{5}+\ldots$? (List all that apply.)

I. $\sum_{k=1}^{\infty} \frac{k!}{k+1}$ II. $\sum_{k=0}^{\infty} \frac{k!}{k+1}$ III. $\sum_{k=2}^{\infty} \frac{(k-1)!}{k}$

Part III - Parametric and Polar Graphs.

5. the graph of $r = 4\sin(2\theta)$ is drawn below. For what values of θ is the graph drawn in quadrant IV?



6. (Calc) An object is fired from ground level. The initial velocity, v_0, is 110 ft/sec. the angle at which the object is fired is $\frac{\pi}{3}$ with the horizontal. The following equations describe the path of the projectile where t is time in seconds and g is acceleration due to gravity (-32 ft/sec/sec). What is the range of the projectile (how far away will it land)?

$$x = (v_0 \cos\theta)t$$

$$y = \frac{1}{2}gt^2 + (v_0 \sin\theta)t$$

7. A laser pointer is moving in the xy-plane with the following coordinates:

$x = 3\cos\theta$

$y = 4\sin\theta$ for $0 \le \theta \le 2\pi$

a. What are the coordinates of the point when $\theta = \frac{5\pi}{6}$?

b. What is the rectangular equation containing not trigonometric functions that represents the graph of the set of parametric equations?

c. The slope of any point on the curve can be found using $m = \frac{4\cos\theta}{-3\sin\theta}$. Write a rectangular equation for the tangent line at the point where $\theta = \frac{5\pi}{6}$. Do not include trigonometric functions in the equation.

Answers to Summer Assignment

1. $15x^2+15xh+5h^2$	2. $\frac{3x^2-4x-5}{(x-1)^3}$	3. $\frac{b-b^2}{b^2+a}$	4. $\frac{2x(x+1)-3(x+1)^2}{2x(4x+3)}$
5. $x=-\frac{5}{3}$	6. $x=-\frac{1}{2},-2,-1$	7. $y_1=\frac{3+xy}{xy^2-x-5}$	8a. $b>a$
8b. $a>b$	9. $1+\frac{1}{u}$	10. $1+u^{-\frac{1}{6}}+u^{-\frac{1}{2}}$	11. $5x^{\frac{5}{2}}-2-4x^{-\frac{1}{2}}$
12. 4	13. 2	14. -16	15. hor shrink of $\frac{1}{2}$
16. hor stretch of 2	17. reflect over y-axis	18. same graph	19. $g(x)=f(x)-3$
20. $f(x-4)-3$	21. $g(x),h(x)$	22. $\lvert f(x)\rvert+2$	23. [-3, 1]
24. $x=2.148$	25. $f(x)=-2\lvert x\rvert+5$	26. min is -3	27. $x=-1,8.5$
28a. $g(t)=360-2t$	28b. $g(20)=320$ gal left in tank	28c. $f(t)=\begin{cases}360-2t & 0\le t\le 20\\ 320+4t & t>20\end{cases}$	29b. D: all real; R:[$-\infty$,2]
29c. -8	29d. -2, -4	29e. $f(x)=\begin{cases}x+1 & x\le -3\\ -x-5 & x>-3\end{cases}$	30b. 9, 1, undefined
30c. not continuous	31a. $\frac{5}{12}$	31c. undefined	32a. (0, A)
32b. (F, I)	32c. (A, F)	32d. (D, E)	33a. $x=-1$
33b. $x=-1$	33c. $y=3$	34. -7	35. $f(x)=\frac{-(x-1)^2}{(x+3)(x+1)}$
36. -.75	37. $a=16;b=3;c,d=9,-2$	38. $y=x+1$	39. (3, 5)
40. $\frac{1\pm\sqrt{5}}{2}$	41. -6.25	42. I, III	43. $y=3x+7$
44. check on calc	45a. 4	45b. 0	45c. 1
45d. 0	45e. 1	45f. 2	46. $f^{-1}(x)=\frac{\ln x+2}{2}$
47. II, III	48. $9(3^b)$	49. check on calc	50. (-.25, -.368)
51. (3, 2) (-3, -2)	52. $\left(-13,-\frac{1}{3}\right)$	53. $\frac{12\sqrt{2}-12}{\pi}$	54a. $\frac{1}{\sqrt{3}}$
54b. $\frac{1}{2}$	54c. 0	54d. $\frac{\pi}{3}$	54e. $\frac{\pi}{3}$
54f. $-\frac{\pi}{3}$	55. check on calc	56. even: cosine, secant Odd: sine, cosecant, tangent	57. [-3, -1]
58. $\frac{7\pi}{6},\frac{11\pi}{6}$	59. (3.544, 9.022)	60. $\frac{\pi}{2}$	61. check on calc
62. 0.005	63. $\left(\frac{\pi}{2},1\right)$	64. $\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}$	65. work may vary

BC Part Only

1a. 16 wolves	1b. 88 wolves (max sustainable population)	1c. 2012	2. I. no II. yes III. no
3. 27	4. I, III	5. $\left[\frac{\pi}{2}, \pi\right]$	6. 327.466 feet
7a. $\left(\frac{-3\sqrt{3}}{2}, 2\right)$	7b. $\frac{x^2}{9}+\frac{y^2}{16}=1$	7c. $y-2=\frac{4\sqrt{3}}{3}\left(x+\frac{3\sqrt{3}}{2}\right)$	